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PRESTRESSED CONCRETE BRIDGE MEMBERS

PROGRESS REPORT NO. 15

FATIGUE RESISTANCE OF PRESTRESSED CONCRETE BEAMS IN BENDING

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Prestressed Concrete
Bridge Members

Progress Report 15

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by

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FRITZ ENGINEERING LABORATORY
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FATIGUE RESISTANCE OF PRESTRESSED CONCRETE BEAMS IN BENDING

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SYNOPSIS

A method is presented to predict the fatigue strength of prestressed concrete based on the failure envelope of the materials involved. A discussion of factors influencing the fatigue strength, such as percentage of steel, level of prestress, and ratio of dead to live load is also included. Recommendations are given on the notion of safety and on the safety factors for prestressed concrete under fatigue loading.

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THE FATIGUE RESISTANCE OF PRESTRESSED CONCRETE BEAMS IN BENDING

INTRODUCTION

A survey of the present status of research in fatigue of prestressed concrete members leaves a rather perplexing impression. Various authors have conducted and reported numerous tests and drawn conclusions which are difficult to compare, because of the complexity of the problem.

A wide-spread opinion that prestressed concrete is safe against fatigue failure, a belief probably stemming from the early tests of Freyssinet,⁽¹⁾ is confronted by a reluctance on the part of many engineers to use prestressed concrete where fatigue loading exists. The existence of these two notions side by side is not surprising when one considers the large scatter of dynamic safety factors from as low as 1.15 reported by Leonhardt⁽²⁾ to as high as 2.60 reported by Lin.⁽³⁾ Other authors mention ratios of dynamic ultimate strength to static ultimate strength of 0.40 (Inomata)⁽⁴⁾ to 1.00 (Abeles).⁽⁵⁾

On the basis of the above facts, it seems imperative that the causes for such wide variation should be investigated theoretically. This gives rise to the question of whether or not it is possible to predict the dynamic ultimate strength

of a member with about the same degree of accuracy which can be achieved in calculating the static ultimate strength.

The literature offers little help in answering these questions, and therefore it is the primary purpose of this paper to present a method for the calculation of the dynamic ultimate loads which permits an adequate consideration of the safety of a member in bending. The method which is presented here has been used successfully in checking the results of several investigations, including those of Lehigh University.

PREDICTION OF THE BEHAVIOR OF BEAMS UNDER DYNAMIC LOADS

The Stress-Moment Diagram

The first information required is the stress-moment relationship for a critical cross-section of the beam in question. In order to make this relationship as general as possible, the ordinate and the abscissa are non-dimensionalized by expressing the actual stresses and moments as a percent of the corresponding ultimate stresses and moments (Fig. 1).

The uppermost curve gives the theoretical relationship between steel stress and total applied moment, from release of prestress, through cracking, and up to the static ultimate moment. In the elastic range, i.e. before cracking, the steel

stresses increase only very little and at cracking load there is theoretically a sudden break. In reality there is no such discontinuity, since the cracks develop gradually. For theoretical considerations, however, it is convenient to assign the total stress increase due to cracking to one moment -- the cracking moment. The curves in this example are drawn for a beam having a percentage of steel required for balanced design. A design is called "balanced" if the stress in the steel at the ultimate moment corresponds to the stress causing 0.2% permanent set in the steel. The stress-moment relationship for the concrete top fiber starts out as a straight line, and gradually becomes concave downward as static ultimate load is approached. The bottom fiber stresses, on the other hand, increase more or less linearly to their terminal point at the initial cracking moment. Such a diagram is characteristic for a given cross-section and a given effective prestressing force, but it does not depend on the location of the cross-section, because the dead load is included in the external bending moment. Hence for a beam with straight tendons and constant moment of inertia the above diagram is valid for any cross-section. It will be noted that the requirements of the Bureau of Public Roads Criteria⁽⁶⁾ were observed in laying out these curves, in that, the steel stress under working load is at 60% of ultimate

for steel, and the maximum tensile and compressive concrete stresses are respectively 8% and 40% of static ultimate for concrete.

An accurate determination of the stress-moment diagram after cracking is, admittedly, not an easy task for two main reasons. First, the formulation of a strain compatibility condition for a cracked cross-section leads to some difficulty because of the initial state of strain of the uncracked cross-section imposed by the prestressing. Second, the stress-strain relationship of steel and concrete cannot any longer be taken as a straight line. The difficulty with the usual methods of equilibrium and compatibility condition can be overcome by following a method first proposed by Colonetti:⁽⁷⁾

Any external moment M (including the dead load moment) can always be split into a couple, say

$$M = bT = hc$$

$$T = C$$

For example, one could arbitrarily choose T to be the initial prestressing force minus the losses due to creep and shrinkage.

$$T = T_{\text{initial}} - \Delta T_{\text{creep}} - \Delta T_{\text{shrinkage}}$$

(Note that the elastic losses are not subtracted.)

If this T is imagined to act alone on the centroid of the prestressing tendons, it would just eliminate all the concrete stresses. The actual concrete stresses can thus be calculated

under the action of the eccentric compressive force C alone by means of the usual methods of equilibrium and compatibility.

(The force C can very well lie outside the cross-section; if M is large enough, the lever arm of the external forces

$$h = \frac{M}{T} = \frac{M}{C}$$

can become larger than the effective depth of the beam.) The steel stresses, in turn, can be found by superimposing the action of T and C . However, since this involves several steps by trial and error, and furthermore, requires a definite knowledge of the stress-strain relationships of concrete and steel, the method remains somewhat lengthy.

The Fatigue Failure Envelope for Prestressing Steel and Concrete

Thus far only the static properties of the critical cross-section have been dealt with. In order to find a connection to the dynamical behavior, two fundamental diagrams which show the dynamic properties of steel and plain concrete are introduced. It is customary in strength of materials to present this information as a failure envelope for a given fixed number of cycles. It is beyond the scope of this paper to extensively deal with the question of how large to choose this number of cycles - a question which involves statistical considerations of the expected load frequency as well as consideration of the dynamic characteristics of the material itself.

We only mention that most of these materials show a more-or-less distinct fatigue limit, i.e. above a certain number of cycles, which can vary from one to several millions, the ultimate capacity levels asymptotically to a fixed value. For our combination of concrete and high tensile steel it seems to be justified, although not beyond critical consideration, to restrict the discussion to a fatigue limit assumed at one million cycles.

The fatigue failure envelope for prestressing steel, shown in Fig. 2, is a modification of the well-known Goodman-Johnson diagram.⁽⁸⁾ This envelope indicates how much we can increase the stress from a given lower level to obtain a failure at about one million load-cycles. (Again the stresses are expressed as a percentage of static ultimate strength.) The higher this lower stress limit is chosen, the smaller becomes the possible stress amplitude. Thus, the steel may resist a repetitive range of stress amounting to 27% of static tensile strength if the lower stress limit is zero, but only an 18% range of stress if the lower stress limit is increased to 40%. The trend of possible stress ranges may be easily observed by noting the vertical displacement and width of the shaded area. At the point where the abscissa is 40%, for example, it will be seen from the vertical axis that the shaded area extends

from 40% to 58%. Furthermore, it may be seen that if the stress were increased so that the minimum stress limit was 80%, the shaded area extends to only 85%, thus giving a possible stress range of only 5%.

The failure envelope in Fig. 2 readily provides a definition of the dynamic yield point for materials with no distinct flow point. It can be shown by tests that there exists a stress level smaller than the static ultimate strength above which even an extremely small stress alternation will cause dynamic fracture. This stress level, at about 95% of the ultimate, shall henceforth be defined as the dynamic yield stress.

A failure envelope analogous to the one for steel can be found for concrete (Fig. 3). This curve is more complicated because it must be drawn for a range covering both tensile and compressive stresses.⁽⁹⁾ For example, if a point is considered where a tensile stress range occurs, the portion of the envelope in the upper right quadrant adjacent point A will apply. If a point, such as the bottom fiber of a prestressed beam is being considered, it might readily be visualized that the fiber may be stressed between, say, 20% compression and 10% tension, for one million cycles before cracking took place. The point on the envelope representing a possible range of

stress between 20% compression and 10% tension is denoted by the point B. The third quadrant contains the portion of the failure envelope representing compressive stress ranges. For example, a compressive stress could be applied between the limits of 40 and 80% (as denoted by point C) for one million cycles before failure. The possible stress amplitudes are again accentuated by the shaded area in the same way as for the steel failure envelope. Apparently there exists also a dynamic yield stress in concrete approximately at 90% of the static ultimate strength.

The Combined Diagram

Returning now to the problem of predicting the dynamic ultimate strength in bending of a critical cross-section, we simply combine the three previous diagrams, with the stress-moment relation as the central part, and the steel and concrete failure envelopes plotted for reasons of clearness, to the left and to the right. Figure 4 shows the resulting combined diagram, which makes possible the determination of the dynamic cracking load, and the dynamic ultimate moment as limited by steel or concrete.

Determination of the Dynamic Cracking Moment

Several simple steps are necessary in the determination of the dynamic cracking load. In Fig. 4, point A on the

stress-moment diagram represents the bottom fiber stress at dead load moment and is the starting point from which a horizontal projection is made over to the failure envelope of concrete. Having a lower stress limit at point B on the failure envelope, it is only necessary to project vertically to point C and establish the upper limit of the stress range which causes cracks after 1 million cycles. A horizontal projection from point C back to an intersection with the bottom fiber curve on the central diagram at point D results in the establishment of the dynamic cracking moment at 40% of static ultimate moment. In other words, in this particular example, 1 million load cycles from the dead load moment to 40% of the static ultimate moment will induce cracks in the bottom fiber of concrete.

Determination of the Dynamic Ultimate Moment

The ultimate moment under dynamic loading is found in the same manner as the cracking load, i.e. by projecting in turn from the moment-stress diagram to the failure envelope and back. In order to obtain the minimum failure moment, however, we have to observe two failure conditions - one for steel and one for concrete. The dynamic ultimate moment based on the steel is 56% of static ultimate as found by projecting from E to F to G and finally to H. The dynamic ultimate based on the top fiber of concrete is determined by projecting from I to J to K to L and is 83% of the static ultimate moment.

Thus, the dynamic ultimate moment of the beam for which Fig. 4 was drawn is limited by the steel, and is 56% of static ultimate moment.

DISCUSSION

The method described above provides an adequate means for determining the dynamic capacity of a prestressed concrete member in bending. It is therefore possible to discuss the effect of the level of prestress, the effect of over- and under-reinforcing, and the cracking characteristics.

Level of Prestress

In Figure 5 the stress moment relationships are plotted for different levels of prestressing. Imagine three beams which are identical except for the fact that the steel in the first beam has no initial prestress, the steel in the second beam has a prestress which is 30% of its static ultimate strength, and the third beam has steel with 60% initial prestress. The moment-stress diagrams for the beam with zero initial prestress is represented by the solid lines, for 30% initial prestress by the dashed lines, and for 60% initial prestress by the broken lines. For zero prestress the maximum possible stress range in the steel is seen to be from zero to 30%, thus giving a dynamic ultimate moment of only about 27% of static ultimate

moment by projecting from A to B to C to D. Correspondingly, it can be found by projecting from E to F to G to H that the beam with 30% initial prestress has a dynamic ultimate moment based on the steel of 42% of static ultimate moment. The highest dynamic ultimate moment results from the beam with the highest initial prestress of 60% of its static tensile strength (projecting from I to J to K to L) and this value is 52% of static ultimate moment.

The values of dynamic ultimate bending moment obtained above for various levels of prestress indicate that under otherwise identical circumstances these values increase with the increasing level of prestress. This statement has been confirmed by many tests, the first having been performed by Freyssinet⁽¹⁾ in 1934. It should be kept in mind, however, that the concrete stresses are assumed to be not critical.

Over- and Under-reinforcing

Fig. 6 shows the stress-moment relation for three beams which are identical except for the amount and arrangement of the prestressing steel. The first beam, for example, has a small percentage of steel and is under-reinforced. The second beam is assumed to be a balanced static design on the basis of the definition previously stated in this paper. The third beam has more steel than is needed for a balanced design and is therefore over-reinforced.

The dynamic ultimate moments for all three beams based on the steel may be found quite easily. It will be observed that the possible stress range, denoted by the line B - C on the failure envelope for the steel is practically common to all three beams because of the close correlation of the stress-moment curves over the elastic range. The dynamic ultimate moments based on the steel are observed to be the abscissas of the points D, E, and F for the under-reinforced, the balanced, and the over-reinforced designs, respectively.

It is evident from an examination of the relationship between the stress-moment curve G - J and the failure envelope for the concrete, that the top fiber of the concrete does not become critical unless there is a percentage of steel which is larger than the balanced design percentage for static loads. In other words, there exists a balanced design percentage of steel for dynamic loading which results in an optimum dynamic ultimate moment. In Fig. 6, this optimum dynamic ultimate moment is approximately 85% of static ultimate moment.

It has been observed that the variation of the percentage of steel, within the range of practicable values, can result in a wide range of variation of the ratio of dynamic ultimate moment to static ultimate moment. It follows that probably the higher the percentage of steel in any given cross-section, the higher will be the resistance to dynamic loading,

but the arrangement of steel must also be considered. In fact, it appears to be advantageous from the point of view of fatigue loading to use a percentage of steel higher than the percentage for static balanced design, that is, over-reinforce the section. It is known that in most cases the steel, not the concrete, is the critical material in fatigue, even though statically the section might fail by crushing of the concrete in the top flange. The amount of increase in steel stress at cracking of the bottom fiber is inversely proportional to the percentage of steel in the section. Furthermore, the slope of the curve becomes flatter with increased steel percentages, and hence leads to an advantage in over-reinforcing for fatigue conditions.

There has always been an aversion against using over-reinforced concrete members of any kind. The principal reason for this seems to stem from the fact that over-reinforced members usually fail without warning, and before any large deflections are observed. However, in the case of failures caused by dynamic loading, failure can occur suddenly, whether the beam is over- or under-reinforced, and before deflections become large, moreover, the chance that the strands or bars do not fail simultaneously increases with the number of tendons, thus again favoring over-reinforcing. Also, the failure can be sudden, no matter whether it occurs in concrete or steel.

Cracks

One fact, which is quite apparent from a study of Fig. 4 is that the dynamic cracking load will always be less than the static cracking load. However, this should not be regarded with too much concern by the engineer.

A recent investigation conducted at the Fritz Engineering Laboratory, consisting of static and dynamic testing of two full scale pretensioned bridge members, tends to substantiate the statements in the preceding paragraph.⁽¹⁰⁾ It was found that the stiffness and general behavior of the dynamically tested beam compared quite favorably with the second beam tested by a gradually applied load. Observations revealed that whereas cracks in the dynamically loaded beam occurred below static cracking moment, their number and width at higher moments compared very favorably with values for the statically tested beam.

Further study of Fig. 4 reveals that the ratio of dead load moment to live load moment has very little effect on the dynamic cracking moment (bottom fiber). For example, a ratio of dead to live load of zero will result in a theoretical cracking moment of about 40% of static ultimate moment, whereas a ratio of five increases the cracking moment to only about 43% of static ultimate moment.

Although repetitive loading necessarily reduces the cracking load, the advantage of prestressing remains the same in that it delays the occurrence of cracks very substantially.

Notion of Safety

Now that a means of predicting the dynamic strength of prestressed members is established, it seems justified to discuss the notion of safety in order to arrive at a suitable means of providing sufficient safety against the probability of failure.

Since the actual stresses due to prestressing do not increase linearly with the applied moment, it was felt that merely specifying allowable stresses is insufficient. Thus an additional restriction is imposed on the external moment by all the current codes.

Formulas for limiting the maximum allowable bending moments are as follows:

$$M_{ult} \geq K_1 (D + L + I)$$

$$M_{ult} \geq D + K_2 (L + I)$$

When: D = dead load moment
 L = live load moment
 I = impact moment
 K_1, K_2 = load factors*

The nature of these formulas often leads to the idea that they provide a margin of safety against overloading. Such an idea

*As discussed later, K_1 (but not K_2) can be considered as a safety factor.

is both erroneous and dangerous. The necessity for safety factors has quite a different reason, namely, it stems from the fact that the prediction of the ultimate moment always includes a number of uncertainties which can be summarized, according to Rüsch,⁽¹⁰⁾ as follows:

1. Uncertainties in the calculation of the internal forces and stress.
2. Uncertainties in the properties of the materials (concrete and steel).
3. Local weaknesses of the materials (segregations in concrete and inclusions in steel).

The real purpose of the factor of safety should be to guarantee a sufficiently small probability of failure under working load assuming that an unlucky coincidence of malpredictions due to the above uncertainties might lead to a failure under working load. Two points are significant, first, that a possible failure is not excluded but only made highly improbable, and second, that this probability refers only to a given load - usually taken to be the maximum working load. The probability itself can be obtained by a theoretical evaluation of statistical surveys of simple tests.

Since these uncertainties are fully independent of the nature of the load, it follows that dead and live loads

have to be dealt with in the same manner. A formula of the kind

$$M_{ult} \geq D + K_2(L + I)$$

is therefore notionally unsatisfactory although it is clear in its intentions. It might be mentioned, incidentally, that the notion of allowable stresses is clear in this respect, because nobody ever suggested that one should distinguish between allowable stresses due to live and due to dead loads.

If there are believed to be some uncertainties in the live and impact load assumptions, then it is necessary to increase these load assumptions, but the safety factor should not be used to take care of overloading since this would simply mean that the required safety margin would no longer be guaranteed. It seems especially desirable for short bridges to assume occasional overloading, thus eliminating the need for any restrictions of the above type.

From all the foregoing it follows that the factor of safety should clearly be separated from the overloading capacity. The former is connected with the uncertainties of our calculation and assumptions while the latter is a definite value and can only be determined by the actual destruction of the structure. Only in two cases does the safety factor coincide with the overloading capacity; if the deviations

from our assumptions are all zero (which is never the case), or if the underestimations are accidentally equalized by overestimations.

If a given bridge should, during its life, sustain permanently larger loads than those for which it was designed, then a careful consideration of the actual failure probability can always decide whether or not such overloading can be tolerated.

All these remarks apply to dynamic as well as static loading. Since the uncertainties under the latter are not smaller for one than for the other, the design moment should be subjected to a restriction of the form

$$M_{\text{dyn-ult}} \geq K(D + L + I).$$

As to the actual value of the safety factor K , this might very well be a matter for further discussion, but it is believed that it should be somewhere around two.

Many engineers will argue that in an actual bridge 1 million cycles of the maximum load will be very unlikely; this being true under certain conditions, we can only see one way of taking into account such possibilities - that is, by a statistical proof that the expected maximum load frequency minimizes the danger of fatigue failure. Such proof would certainly be possible for many highway bridges, but probably not for railway bridges.

Another widespread argument against considerations of dynamic failures is based on the fact that the steel-stress variation is definitely small and that the corresponding failure envelope leaves a margin of say three times as large a stress variation. From all the foregoing, it follows that this factor of three cannot be considered as a factor of safety. Certainly no one would pretend that the safety factor of any structure is two if only the allowable stresses are taken to half of the ultimate stresses. The above argument, however, would amount to an analogous, misleading contention.

The foregoing discussion of the desirable requirements on prestressed structures under repetitive loading may seem rather pessimistic. To leave such an impression, however, is quite opposite to the aim of this paper. An attempt was made to show that prestressed concrete is very well suited to structures subjected to dynamic loads. The method described in this paper is intended to give more certainty as to what is to be expected under such conditions. Since it is believed that the estimated dynamic capacity can be predicted, it seems only fair to connect this also with sufficient safety requirements.

Although a dynamic safety factor of, say two, may seem very severe, attention is drawn to the fact that fatigue loading, especially if combined with severe exposure conditions,

involves more uncertainties than those mentioned previously for static conditions.

Now investigate the possibility that the present design requirements may provide sufficient safety against fatigue failure. If one considers the two formulas given by the Bureau of Public Roads Criteria⁽⁹⁾ for limiting the design moment based on the ultimate strength of the member, it will be seen readily by considering typical ratios of live load to dead load that the ratio of dynamic ultimate strength to static ultimate strength must be greater than about 0.8 if a factor of safety of two against dynamic ultimate moment is achieved. A beam must be carefully designed based on the method of analysis given in this paper to have so high a ratio of dynamic ultimate to static ultimate strength. Members designed on the basis of only static considerations normally have ratios of dynamic ultimate to static ultimate of 0.4 to 0.8.

The required dynamic factor of safety cannot be achieved economically by making the required static ultimate moment more conservative because designing for a higher static ultimate moment would result in a larger cross-section of member when all that may be required is a similar member with a larger percentage of steel or a better arrangement of steel.

The majority of prestressed bridges will never be in danger of fatigue failure. But if a structure is subjected to severe repetitive loading, the problem of failure deserves adequate consideration. For any specific problem, the dynamic ultimate moment should be calculated by the procedures outlined in this paper.

CONCLUDING REMARKS

Based on the knowledge of the fatigue properties of concrete and prestressing steel and the determination of the stress-moment relationships of prestressed beams, it has been shown that it is possible to predict the fatigue properties of members in bending revealing the following facts:

1. The dynamic ultimate moment is always less than the static ultimate moment and can vary over a very large range.
2. The dynamic cracking moment is always smaller than the static cracking moment. Above static cracking moment, however, the number and width of cracks due to dynamic loading correspond to those due to static loading.
3. The ratio of the dynamic ultimate moment to the static ultimate moment is increased by:
 - a. Increasing the level of prestress.
 - b. Increasing the percentage of steel in a beam.

The design of prestressed members under severe fatigue loading should be based on the determination of the ultimate dynamic moment. Considering the uncertainties always present in the analysis and in the properties of materials, adequate safety should be provided by a restriction of the following kind:

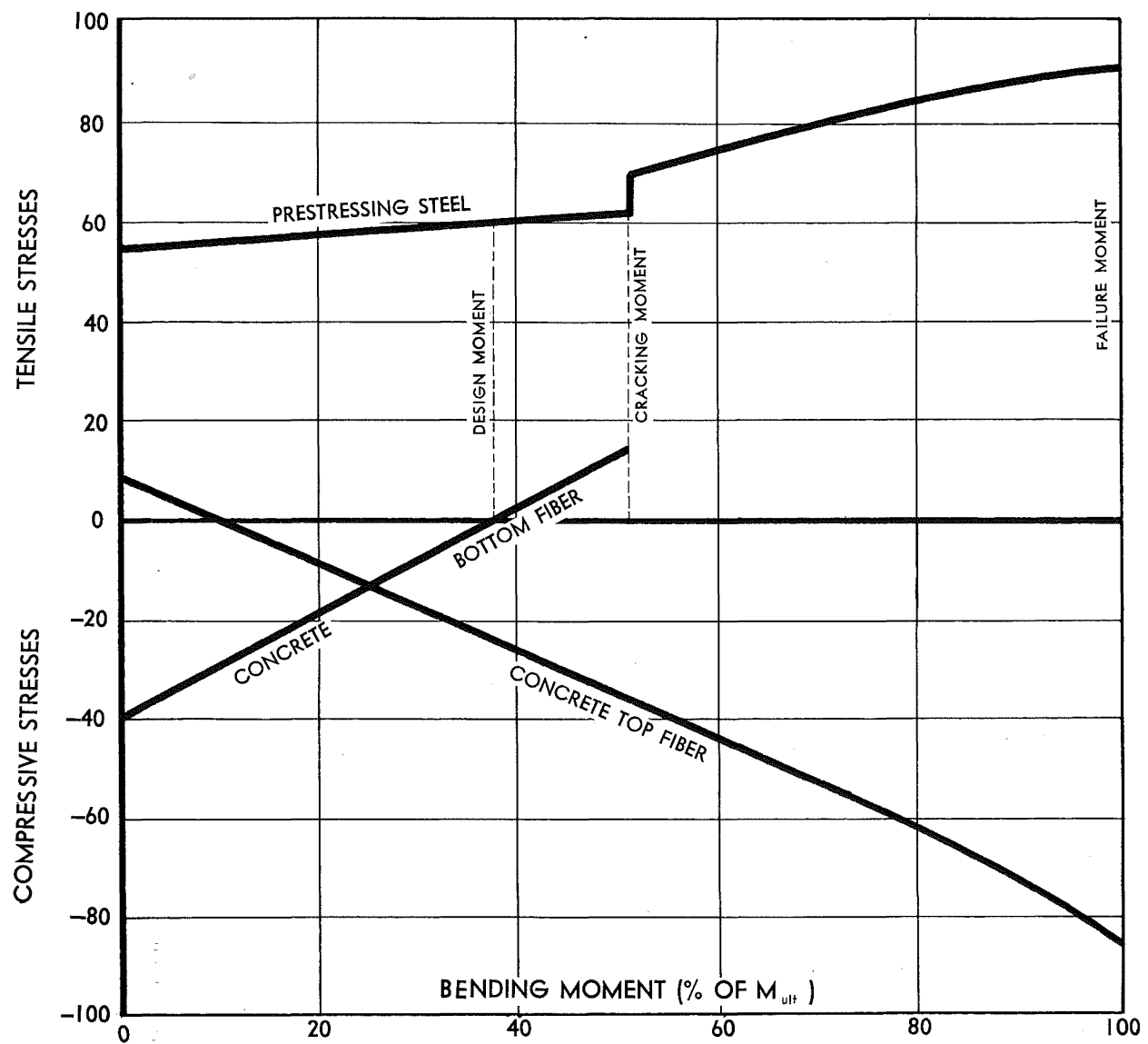
$$M_{\text{dynamic ultimate}} \geq K(D + L + I)$$

The desirable value of the safety factor K (about 2) deserves a careful consideration and should be specified according to the particular type of structure and site conditions. In the case of fatigue loading, it is quite important that possible overloading be considered separately by increasing the design loads and using the above equation with the accepted value of K .

It should be emphasized that this paper is restricted to pure bending of bonded prestressed beams. If the future will, as is hoped, provide more information about the stress distribution under the combined action of moment and shear or if more becomes known about bond stresses, then a further discussion of such failures under repetitive loading will certainly be possible and follow the same pattern as suggested in this paper.

ACKNOWLEDGMENTS

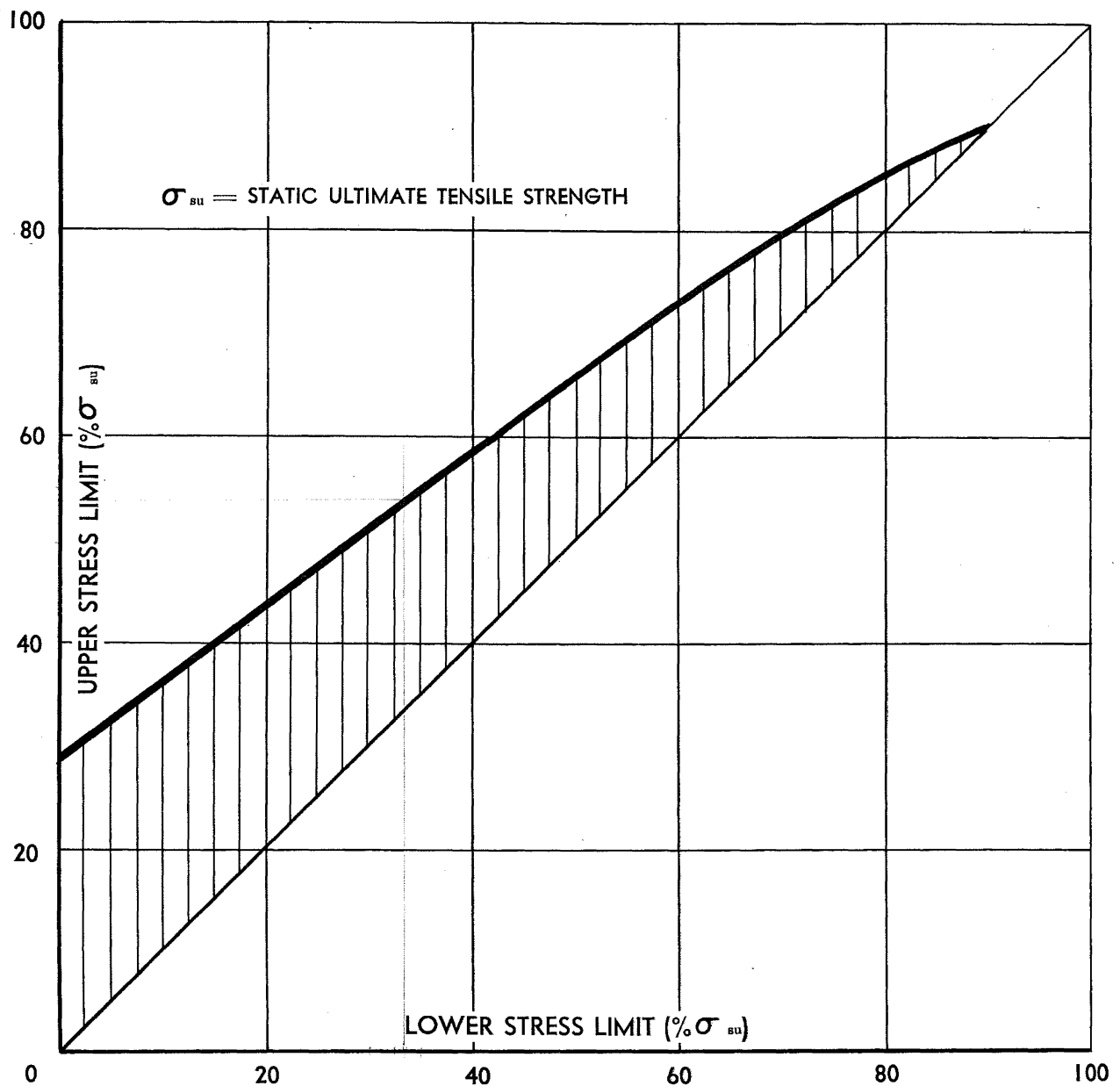
This study has been carried out in connection with research sponsored by the following: Pennsylvania Department of Highways; U. S. Bureau of Public Roads; Concrete Products of America, Division of American-Marietta Company; American Steel and Wire, Division of U. S. Steel Corporation; John A. Roebling's Sons Corporation; Reinforced Concrete Research Council; and Lehigh University.



NOTE: ALL STRESSES IN PERCENT OF CORRESPONDING STATIC ULTIMATE STRESS

STRESS — MOMENT DIAGRAM

FIGURE 1

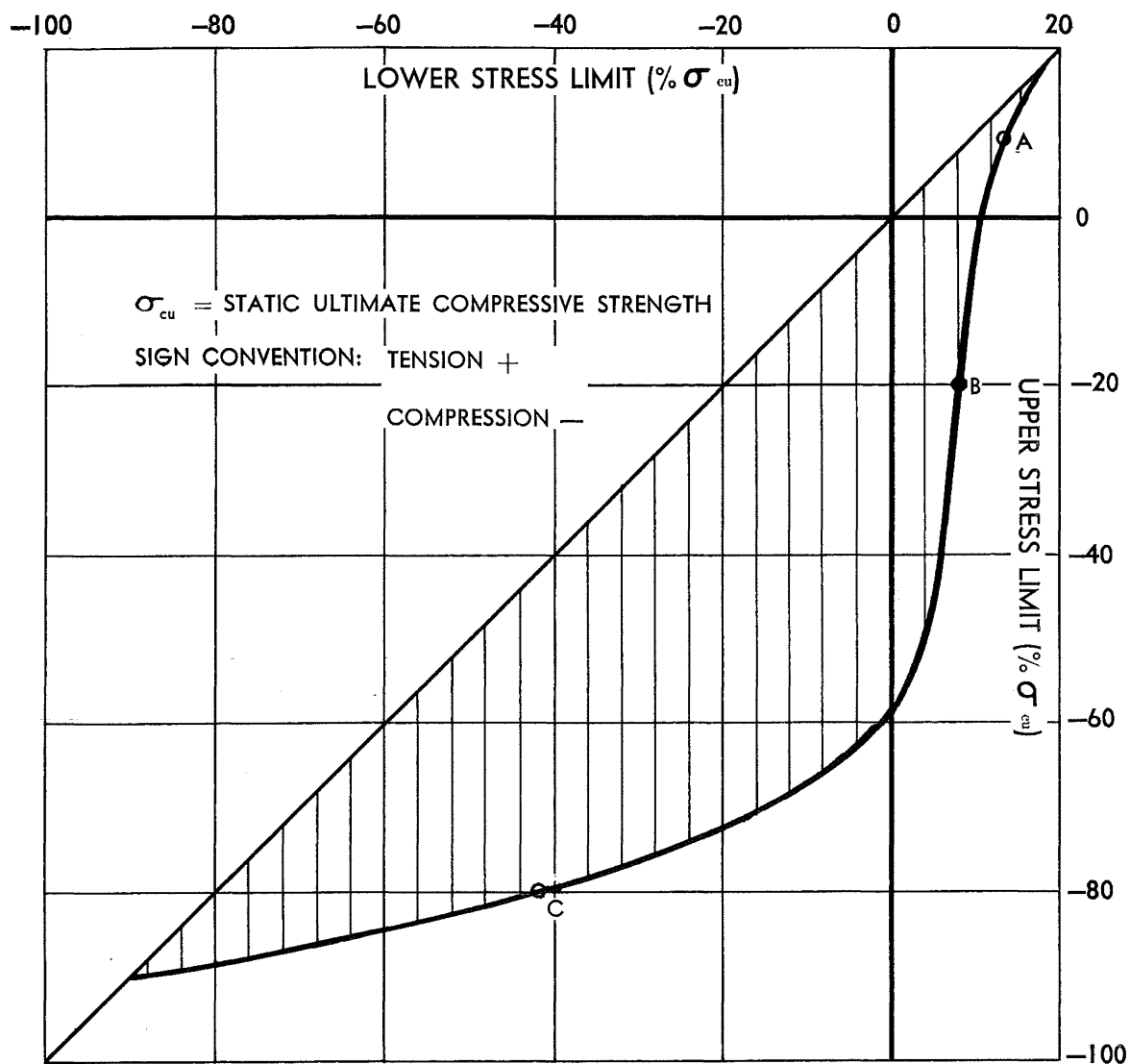


FATIGUE FAILURE ENVELOPE FOR

PRESTRESSING STEEL

(10^6 LOAD CYCLES)

FIGURE 2



FATIGUE FAILURE ENVELOPE FOR CONCRETE

(10⁶ LOAD CYCLES)

FIGURE 3

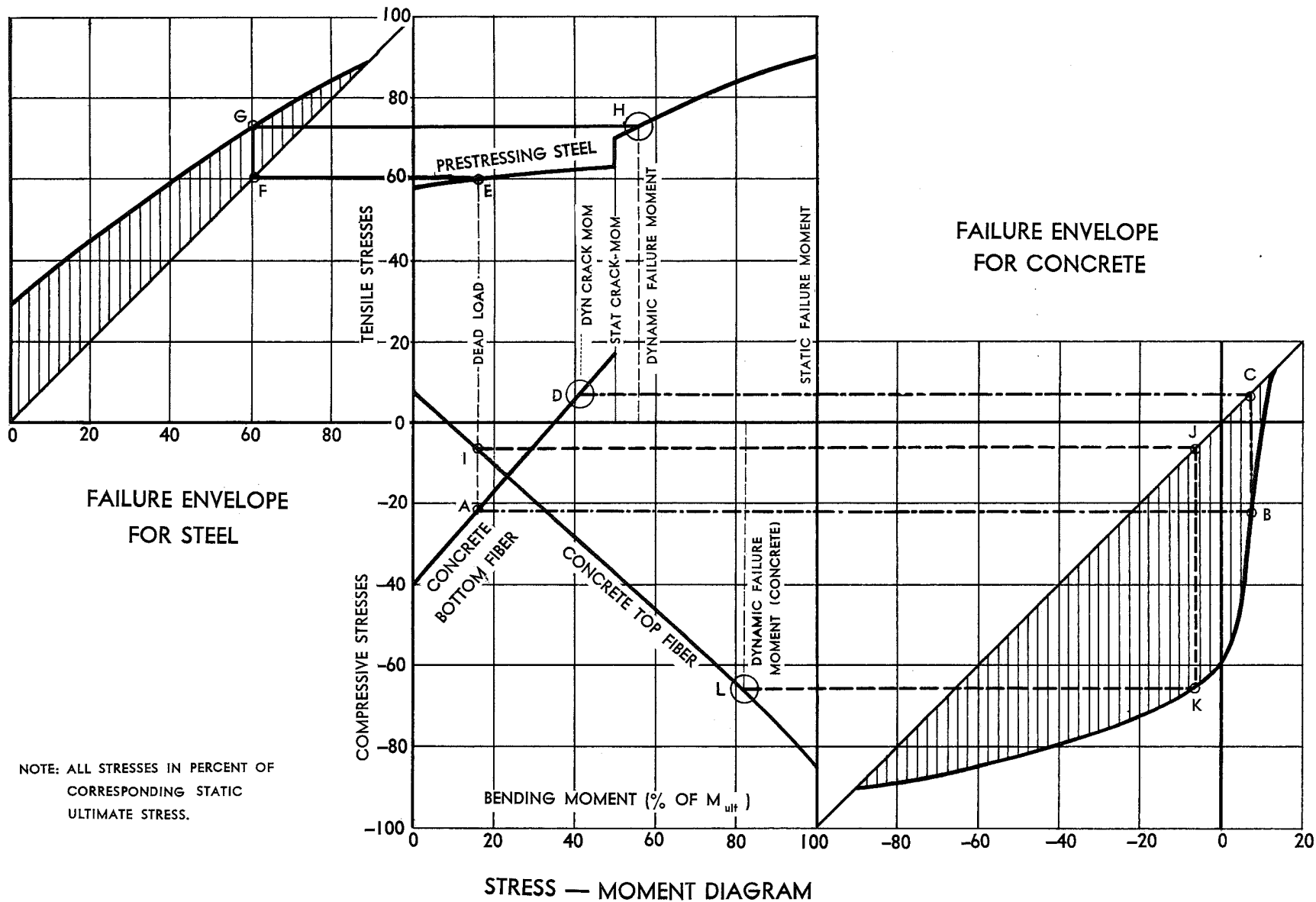


FIGURE 4

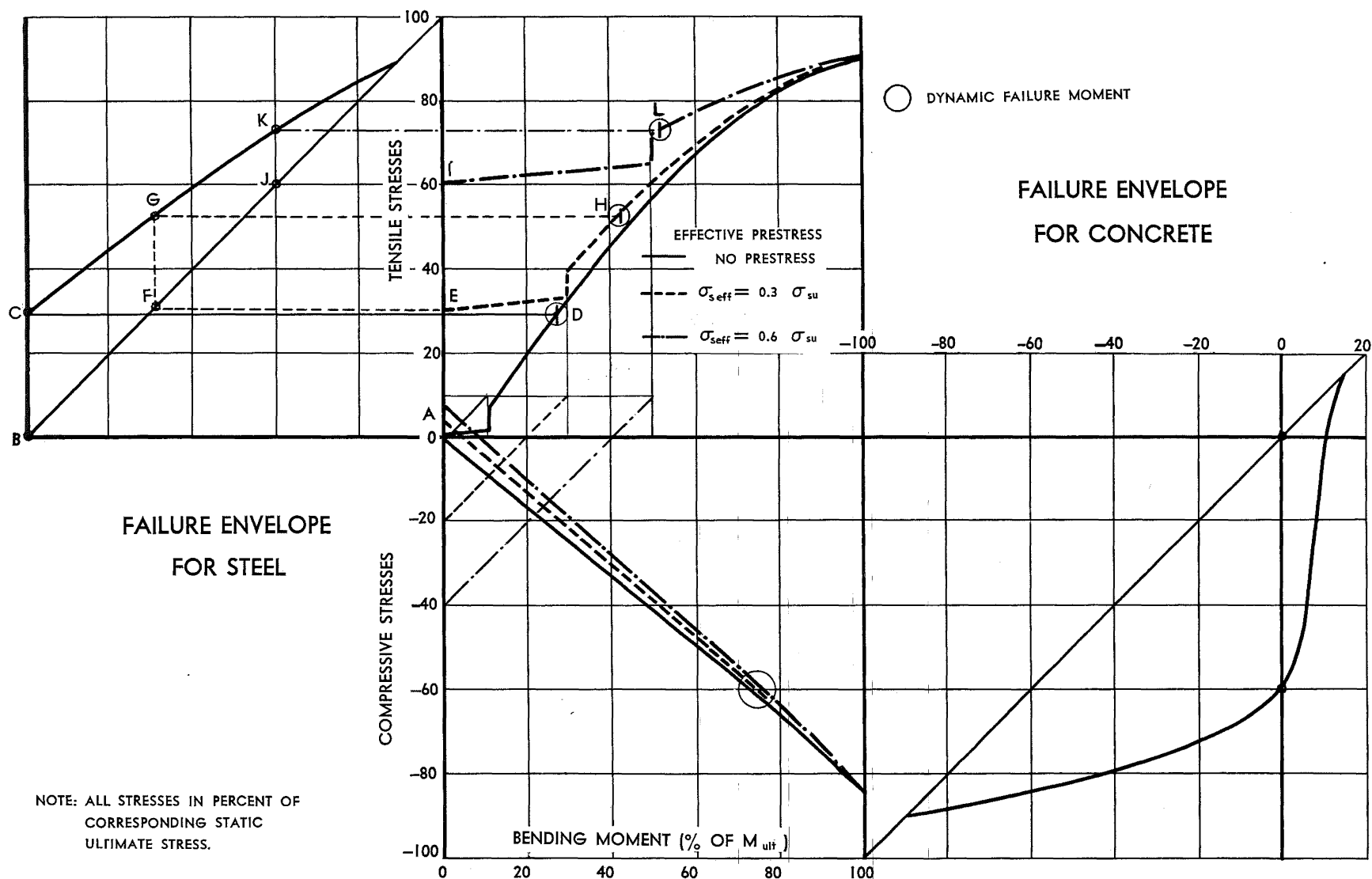
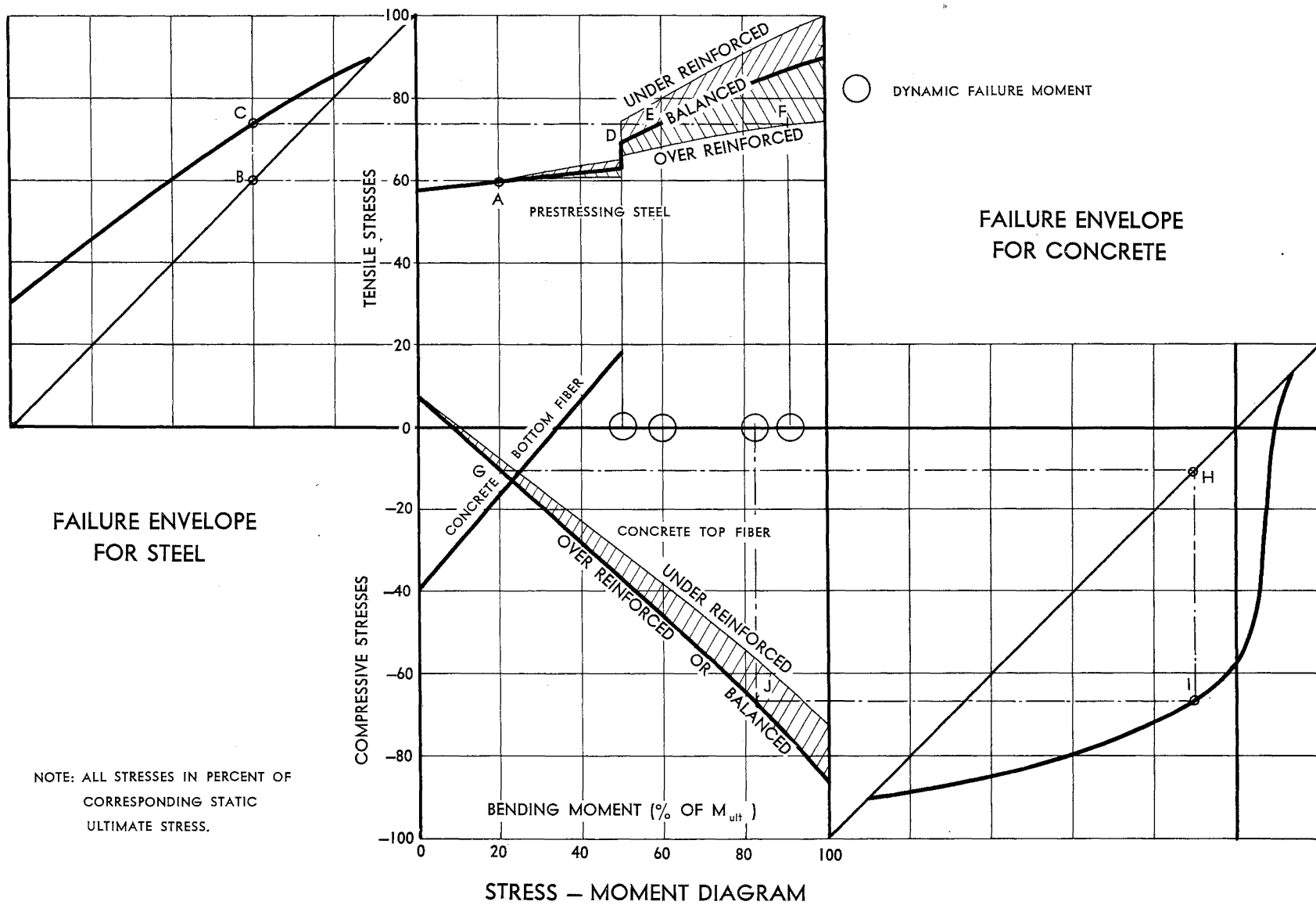


FIGURE 5



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